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Domain walls in five dimensional supergravity with non-trivial hypermultiplets

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Abstract: We study BPS domain wall solutions of 5-dimensional $N = 2$ supergravity where isometries of the hypermultiplet geometry have been gauged. We derive the corresponding supersymmetric flow equations and define an appropriate c-function. As an example we discuss a domain wall solution of Freedman, Gubser, Pilch and Warner which is related to a RG-flow in a dual superconformal field theory.

Keywords: AdS-CFT Correspondance, Supergravity Models.

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Contents

1. Introduction

The AdS/CFT correspondence (for a review see [1] and references therein) relates gauged supergravity in a background geometry $AdS_5 \times H$ to a superconformal field theory (SCFT) in a four-dimensional Minkowski space living on the the boundary of AdS_5. The Kaluza-Klein excitations of the supergravity are identified with gauge invariant operators in the SCFT. H is called the horizon manifold and its isometries are related to the R-symmetry of the superconformal algebra. The original example of [2] considered $H = S^5$ whose isometry group SO(6) corresponds to the R-symmetry of the dual $N = 4$ SCFT. Examples with less supersymmetry and different H-manifolds have been constructed in e.g. [3]–[7].

It is possible to perturb the SCFT by adding an operator O_h to the action

$$S \to S + h \int d^4x\, O_h(x)\,. \tag{1.1}$$

In general this breaks the conformal symmetry and if the perturbation is relevant a renormalization group (RG) flow for the coupling h is induced. The resulting infrared (IR) theory can be free, confining or have a non-trivial fixed point where the β-function vanishes. In the latter case the RG-flow connects two different CFT, the original ultra-violet (UV) theory with the IR theory.

In the dual supergravity description the coupling h is identified with a scalar field Φ and the RG-flow corresponds to a domain wall (DW) solution which interpolates between two different extrema of the scalar potential $V(\Phi)$ [8]–[13]. If the UV-theory and the IR-theory are both conformal the two extrema necessarily have to be AdS_5 vacua of V. The DW solution requires that the scalar field Φ has a non-trivial dependence on the the radial coordinate of AdS_5 which can be identified with the energy scale μ of the RG-flow [12]–[15].

A particular example of such an RG-flow has been presented in [10] and further discussed in [16, 17]. It is a flow from an $N = 4$ SCFT in the UV to an $N = 1$ SCFT in the IR which preserves $N = 1$ supersymmetry along the flow. In the dual supergravity description this was identified with a DW solution of five-dimensional gauged $N = 8$ supergravity connecting two AdS_5 vacua. The solution preserves four supercharges (it is a BPS solution) and interpolates between two extrema one of which preserves the full $N = 8$ supersymmetry and is identified with the UV SCFT while the second extremum only preserves $N = 2$ supersymmetry and is identified with the IR SCFT. The BPS property corresponds to the fact that $N = 1$ supersymmetry is preserved along the RG-flow.

The purpose of this paper is to study BPS domain wall solutions of five-dimensional $N = 2$ gauged supergravity which preserve half of the supercharges ($N = 1$).[1] This establishes the framework for generalized RG-flows which start from an UV theory with less ($N = 1, 2$) supersymmetry. It also simplifies the analysis since the $N = 2$ scalar potential is somewhat less involved than its $N = 8$ counterpart.

An additional motivation arises from the fact that such DW solutions are closely related to a supersymmetric version of the Randall-Sundrum (RS) scenario [18]–[24]. In this scenario gravity is localized near the wall through exponential suppression and therefore requires a DW which asymptotes to IR fixed points on both sides. It has been shown in [25, 26] that there are no such fixed points for theories containing vector/tensor multiplets, but we will argue that this does not necessarily apply if charged hypermultiplets are present.

BPS domain wall solutions of five-dimensional $N = 2$ supergravity have been studied previously. In refs. [27, 28] the DW solutions arising from compactification of 11-dimensional Horăva–Witten M-theory on Calabi-Yau threefolds were derived. In this case the necessity of non-trivial four-form flux results in the gauging of an axionic shift symmetry which is an isometry of the universal hypermultiplet. Refs. [25, 29, 30] considered DW solution with non-trivial vector multiplets and showed that within this setup no IR fixed point can arise. As an immediate corollary also supersymmetric RS domain walls can not be obtained with only vector multiplets [25]. Non-trivial tensor multiplets were considered in refs. [31, 32] but this does not alter the conclusion about possible IR fixed points. Finally ref. [33] derived the five-dimensional gauged $N = 2$ supergravity including vector-, tensor- and hypermultiplets. Many aspects of these discussions go in parallel to domain walls in 4-dimensional gauged supergravity [34, 35].

In this paper we consider both vector- and hypermultiplets and derive the condition for a BPS domain wall solution including charged hypermultiplets. Such a solution is only possible for abelian gauge symmetries. We argue that in this case

[1] We use the convention of $D = 4$ to count supercharges. The minimal supersymmetry in $D = 5$ has 8 supercharges which we call $N = 2$ throughout this paper.

the previous ‘no-go’ theorems do not apply. Specifically, in section 2 we recall a few facts about $N = 2$ gauged supergravity with particular emphasis on gauged isometries in the hypermultiplet sector. It turns out that we have to allow for more general gaugings of the $\mathrm{SU}(2)_R$ symmetry than have previously been considered. In section 3 we study BPS domain wall solutions with both non-trivial vector- and hypermultiplets. We derive the supersymmetric flow equations for the scalar fields and show that the corresponding c-theorem is satisfied. As an application of the formalism we discuss in section 4 the RG-flow of ref. [10]. Section 5 presents our conclusions and contains a preliminary discussion of a smooth supersymmetric RS domain wall.

2. $N = 2$ gauged supergravity

A generic spectrum of five-dimensional $N = 2$ supergravity contains the gravitational multiplet, n_v vector multiplets in the adjoint representation of some gauge group G, n_h hypermultiplets which can be charged under G and tensor multiplets. The gravitational multiplet contains the graviton g_{mn}, two gravitini ψ_m^A, $A = 1, 2$ which are symplectic-Majorana spinors[2] and the graviphoton A_m^0. A vector multiplet contains a vector A_m, two gaugini λ^A and a real scalar ϕ while the hypermultiplet features two hyperini ζ^α and four real scalars q^u.

A tensor multiplet has the same field content as a vector multiplet but with the vector replaced by a tensor. In $D = 5$ vector and tensor are dual to each other and this duality can be performed as long as the tensor fields are neutral under G [31, 32, 36]. In this paper we consider this case and thus only keep n_v vector and n_h hypermultiplets in the spectrum.

The action for scalar fields coupled to supergravity is given by

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2} R - V(\Phi) - \frac{1}{2} g_{MN} \partial_m \Phi^M \partial^m \Phi^N \right] , \tag{2.1}$$

where we have omitted gauge fields and fermions. The Φ^M collectively denote the scalar fields $\phi^i, i = 1, \ldots, n_v$ in the vector multiplets and the scalar fields $q^u, u = 1, \ldots, 4n_h$ in the hypermultiplets, i.e. $\Phi^M = (\phi^i, q^u)$. Supersymmetry forces the metric to be block diagonal

$$g_{MN} = \begin{pmatrix} g_{ij} & 0 \\ 0 & g_{uv} \end{pmatrix} , \tag{2.2}$$

where the metric for the vector multiplets (g_{ij}) has to be very special Kähler [37] and the metric for the hypermultiplets (g_{uv}) has to be quaternionic.

The very special Kähler geometry is best described by introducing $n_v + 1$ functions $X^I(\phi^i), I = 0, \ldots, n_v$ which satisfy one contraint equation

$$\mathcal{V} \equiv \frac{1}{6} C_{IJK} X^I X^J X^K = 1 \, , \tag{2.3}$$

[2] For conventions and notations see [27, 35].

where the C_{IJK} are arbitrary constants determining the scalar manifold. The metric g_{ij} is then obtained via

$$g_{ij} = \partial_i X^I \partial_j X^J \, G_{IJ}|_{\mathcal{V}=1} \,, \qquad G_{IJ} = -\frac{1}{2}\,\partial_I \partial_J \ln \mathcal{V}|_{\mathcal{V}=1} \,. \tag{2.4}$$

The $4n_h$ scalars in the hypermultiplets are coordinates on a quaternionic manifold [35, 33]. This implies the existence of three (almost) complex structures $(J^x)_v^{\ w}$, $x = 1, 2, 3$ which satisfy the quaternionic algebra. Associated with the complex structures is a triplet of Kähler forms $K^x_{uv} = g_{uw}(J^x)_v^{\ w}$ where g_{uw} is the quaternionic metric. The holonomy group of a quaternionic manifold is $\mathrm{Sp}(2) \times \mathrm{Sp}(2n_h)$ and K^x is identified with the field strength of the $\mathrm{Sp}(2) \sim \mathrm{SU}(2)$ connection ω^x_v i.e.

$$K^x = d\omega^x + \frac{1}{2}\,\epsilon^{xyz}\omega^y\omega^z \,. \tag{2.5}$$

As a consequence the K^x are covariantly closed with respect to the SU(2) connection ω^x

$$dK^x + \epsilon^{xyz}\omega^y K^z = 0 \,. \tag{2.6}$$

For later use we need to introduce the quaternionic vielbeins $V_u^{A\alpha}$ defined via

$$g_{uw} = V_u^{A\alpha} V_w^{B\beta} \epsilon_{AB} C_{\alpha\beta} \,, \tag{2.7}$$

where $\alpha, \beta = 1, \ldots, 2n_h$ and $C_{\alpha\beta}$ is the flat $\mathrm{Sp}(2n_h)$ metric.

The isometries on the scalar manifold are generated by a set of $n_v + 1$ Killing vectors $k_I^u(q)$, $k_I^i(\phi)$

$$\delta q^u = \epsilon^I k_I^u(q) \,, \qquad \delta\phi^i = \epsilon^I k_I^i(\phi) \,. \tag{2.8}$$

The k_I^u are determined by a triplet of Killing prepotentials $P_I^x(q)$ via[3]

$$k_I^u \, K_{uv\,B}^{A} = \partial_v P_{IB}^{A} + [\omega_v, P_I]^A_{\ B} \,. \tag{2.9}$$

Furthermore, the P_I satisfy a Poisson bracket relation:

$$\{P_I, P_J\}^A_{\ B} \equiv k_I^u k_J^v \, K_{uv\,B}^{A} - [P_I, P_J]^A_{\ B} = -\frac{1}{2}\, f_{IJ}^{\ K}\, P_{K\,B}^{A} \,, \tag{2.10}$$

where f_{IJ}^{K} is the structure constant of the isometry algebra. It is possible to gauge (part of) these isometries by modifying the covariant derivatives of the scalars, gaugini and hyperini, their supersymmetry transformation and the lagrangian. Furthermore it is possible to (independently) gauge the $\mathrm{SU}(2)_R$ symmetry of the $N = 2$ supergravity. This modifies the covariant derivatives of the gravitino and the gaugino but not the hyperino. Finally, it is also possible to simultaneously gauge the isometries and the R-symmetry. We do not recall the details of this somewhat technical enterprise here but refer the reader to the literature [35, 27, 33].

[3]For SU(2) valued matrices, we adopt the convention $Y^A_{\ B} = iY^x\,(\sigma^x)^A_{\ B}$.

However, for the purpose of this paper, we are lead to consider more general gaugings of the R-symmetry. We modify the procedure outlined in [33], in that we shift the space-time pullback of the SU(2) connection by the following linear combination of the vector fields

$$\omega^A{}_B \to \widehat{\omega}^A{}_B \equiv \omega^A{}_B + A^I \widehat{P}^A_{IB}(q)\,, \tag{2.11}$$

where $\widehat{P}^A_{IB}$ is an SU(2) valued matrix. This differs from the procedure of ref. [33] by the choice of the matrix $\widehat{P}^A_{IB}$. In [33] the Killing prepotential was used i.e. $\widehat{P}^A_{IB} = P^A_{IB}$ while we allow for the possibility of having a slightly more general $\widehat{P}^A_{IB}$ which differs from P_I by a q-dependent scalar (i.e. SU(2) invariant) function

$$\widehat{P}^A_{IB} = \widehat{\gamma}_I(q)\, P^A_{IB} \qquad (\text{no sum on } I)\,. \tag{2.12}$$

A rigorous proof of the consistency of this ansatz is beyond the scope of this paper and we leave it for future investigation. However, as a first check we verified that the new SU(2) curvature $\hat{K}$ defined analogously to (2.5) with ω replaced with $\widehat{\omega}$ satisfies

$$\hat{K}^A{}_B = K^A_{uvB}\mathcal{D}q^u \mathcal{D}q^v + \widehat{F}^I P^A_{I\,B}\,. \tag{2.13}$$

Eq. (2.13) holds in virtue of (2.9) and (2.10), with the definitions

$$\mathcal{D}q^u = dq^u - \frac{1}{2}\widehat{A}^I k^u_I\,, \qquad \widehat{F}^I = d\widehat{A}^I + \frac{1}{4} f^I_{KL}\widehat{A}^K\widehat{A}^L\,, \qquad \widehat{A}^I = \widehat{\gamma}^I(q)A^I\,. \tag{2.14}$$

$\hat{K}$ still satisfies eq. (2.6) with ω replaced with $\widehat{\omega}$.

Thus the only modification following from (2.11) is to replace the gauge fields A^I and their field strength with their hatted counterparts in terms arising from the R-symmetry gauging. As these terms are supersymmetric by themselves [33], this suggests the consistency of our ansatz.[4] For $\widehat{P}_I = P_I$ the consistency has been shown in [33] while for $\widehat{P}_I \neq P_I$ this has not been firmly established yet. The interpretation of the field dependent vector fields $\hat{A}^I$ in the light of the AdS/CFT correspondence will be given at the end of section 3.

Finally, gauging the isometries and the R-symmetry also requires the presence of a scalar potential [35, 27]. Using the modified SU(2) connection, one obtains

$$V = \left(-2G^{IJ} + 4\,X^I X^J\right) \mathrm{Tr}(\widehat{P}_I \widehat{P}_J) + X^I X^J \left(g_{uv}\, k^u_I k^v_J + g_{ij}\, k^i_I k^j_J\right), \tag{2.15}$$

where the first term can be traced back to the gauging of the $\mathrm{SU}(2)_R$ symmetry and the second term to the gauging of the isometries of the scalar manifold. For $\widehat{P}_I = P_I$, this potential coincides with the potential of refs. [35, 27].

[4] We thank A. Ceresole and G. Dall'Agata for discussions on this point.

3. BPS domain wall solutions

In this section we derive BPS domain wall solutions which preserve half of the eight supercharges. As an ansatz for a metric which respects 4d Poincaré invariance we use

$$ds^2 \equiv g_{mn}dx^m\, dx^n = \mu^2(-dt^2 + d\vec{x}^2) + \frac{d\mu^2}{\mu^2\, W^2(\mu)}\,. \tag{3.1}$$

For constant W this is the metric of AdS_5. In the AdS/CFT correspondence the fifth coordinate μ will be identified with an energy scale in the dual four-dimensional field theory [12]–[15]. The UV region (= large length scale in supergravity) corresponds to $\mu \to \infty$, while the IR is approached for $\mu \to 0$. For later use we also note that for this metric the vielbeins and the non–vanishing spin connections are given by

$$e_t{}^0 = e_x{}^a = \mu\,, \qquad e_\mu{}^4 = (\mu\, W)^{-1}\,, \qquad \omega_{t\,04} = \omega_{x\,a4} = \mu\, W\,, \tag{3.2}$$

where $(0, a, 4)$ are the tangent space indices.

We require that the DW solutions preserves four supercharges and as a consequence we have to demand that the supersymmetry variations of the two gravitini $\psi_\mu^A, A = 1, 2$, the $2n_v$ gaugini λ_i^A and the $2n_h$ hyperini ζ^α admit four Killing spinors in the bosonic background specified by the metric (3.1) and vanishing gauge fields. Specifically we demand

$$\delta\psi_m^A \;=\; D_m\epsilon^A - \frac{i}{3}\, X^I\, \widehat{P}^A_{IB}\, \Gamma_m\epsilon^B = 0\,, \tag{3.3}$$

$$\delta\lambda^{Ai} \;=\; -\frac{i}{2}\Gamma^m\partial_m\phi^i\, \epsilon^A + g^{ij}(\partial_j X^I)\, \widehat{P}^A_{IB}\, \epsilon^B \;+ X^I k^i_I\, \epsilon^A = 0\,, \tag{3.4}$$

$$\delta\zeta^\alpha \;=\; -\frac{i}{\sqrt{2}}\, V_u^{A\alpha}\left(\Gamma^m\partial_m q^u \;+ iX^I k^u_I\right)\epsilon_A = 0\,. \tag{3.5}$$

We are mainly interested in the dependence of the scalar fields on the fifth coordinate or, in terms of the dual field theory, on the energy scale μ. Hence we ignore the 4-d spacetime dependence and consider the scalars $\Phi(\mu)$ only as functions of μ.

Let us first consider the (t, x) components of eq. (3.3). They imply a projection on the supersymmetry parameters ϵ^B

$$\left(W\, \Gamma^4\delta^A_B - \frac{2i}{3}\, X^I\widehat{P}^A_{IB}\right)\epsilon^B = 0\,. \tag{3.6}$$

For this to be a consistent projector one learns from $(\Gamma^4)^2 = 1$ that

$$W^2\, \delta^A{}_B = -\frac{4}{9}\,(X^I\widehat{P}_I X^J\widehat{P}_J)^A{}_B\,. \tag{3.7}$$

An additional constraint arises if one demands that (3.6) admits four Killing spinors. This is the case if all SU(2) matrices $\widehat{P}_I$ can simultaneously be rotated in the direction of σ^3 or equivalently if

$$[\widehat{P}_I, \widehat{P}_J] = 0 \tag{3.8}$$

holds.[5]

[5] One could consider $X^I\widehat{P}_I$ as a single q and ϕ dependent SU(2) matrix. However, diagonalizing this matrix is not a covariant operation in this formalism.

In the rotated basis (denoted by the primed quantities) the projector (3.6) reads

$$\Gamma^4 \epsilon'^A + (\sigma^3)^A{}_B\, \epsilon'^B = 0\,, \tag{3.9}$$

while W simplifies to

$$W = \frac{2}{3}\, X^I \widehat{P}'^3_I\,. \tag{3.10}$$

The μ component of eq. (3.3) leads to a first order differential equation which determines the μ dependence of ϵ^A

$$2\mu\, D_\mu \epsilon^A = \epsilon^A\ \ , \tag{3.11}$$

where we used (3.6) and (3.7). The compatibility of this equation with the projector (3.6) will be discussed after solving (3.4) and (3.5) which we turn to now.

Inserting the projector (3.6) into (3.4) yields

$$\left(\frac{1}{3}\mu\frac{d\phi^i}{d\mu}X^I + \partial^i X^I\right)\widehat{P}^A_{IB}\epsilon^B + X^I k^i_I\, \epsilon^A = 0\,. \tag{3.12}$$

Since $\widehat{P}^A_{IB}$ can be rotated into the σ^3 direction the two terms of eq. (3.12) have to vanish independently (unless all supercharges are broken). Thus with our choice of projector the isometries of the vector multiplets cannot be gauged, that is we have to demand $X^I k^i_I = 0$. The vanishing of the first term in (3.12) imposes a first order differential equation for the scalar fields

$$\mu\,\frac{d\phi^i}{d\mu} = -3\, g^{ij}\partial_j \log W\,. \tag{3.13}$$

Finally, inserting the projector (3.6) into the hyperino variation (3.5) yields

$$\mu\,\frac{dq^u}{d\mu} = \frac{2}{3W^2}\,\mathrm{Tr}(X^I \widehat{P}_I\, X^J k^v_J J_v{}^u)\,, \tag{3.14}$$

where we have used (3.7) and the fact that the quaternionic complex structures are given by

$$(J_u{}^v)^A{}_B = g^{wv}(K_{uw})^A{}_B = i\, V^{A\alpha}_u\, V^v_{B\alpha}\,. \tag{3.15}$$

Rewriting eq. (2.9) as

$$X^I k^u_I (J)_u{}^v = g^{vu}\, D_u\,(X^I P_I)\,, \tag{3.16}$$

we can recast (3.14) in the form

$$\mu\,\frac{dq^u}{d\mu} = \frac{2}{3W^2}\, g^{uv}\,\mathrm{Tr}\left(X^I \widehat{P}_I\, D_v\,(X^J P_J)\right)\,. \tag{3.17}$$

Eq. (3.17) can also be written as a gradient flow

$$\mu\,\frac{dq^u}{d\mu} \ = \ -3\, g^{uv}\partial_v \log W \tag{3.18}$$

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with the same W as in (3.13), provided

$$\mathrm{Tr}\left(X^I \widehat{P}_I \, D_v \left(X^J P_J\right)\right) = \mathrm{Tr}\left(X^I \widehat{P}_I \, D_v \left(X^J \widehat{P}_J\right)\right) \tag{3.19}$$

holds. This is a non-trivial constraint on the scalar function $\widehat{\gamma}_I(q)$ of eq. (2.12) and is solved by

$$\widehat{\gamma}_I(q) = 1 - \lambda_I \left(\det P_I\right)^{-1/2}, \quad (\text{no sum on } I), \tag{3.20}$$

where the λ_I are arbitrary constants. For $\lambda_I = 0$ one has $\widehat{P}_I = P_I$ and (3.18) holds without any further condition. However, eq. (3.20) shows that also for non-trivial $\widehat{\gamma}_I$ or in other words for a $\widehat{P}_I$ which differs from P_I the differential constraint on the hyper-scalars is of the form (3.18). We will see in the next section that non-trivial $\widehat{\gamma}_I$ are crucial in order to recover the RG-flow of ref. [10].

Finally the compatibility of eq. (3.6) with (3.11) imposes the additional condition

$$D_\mu (W^{-1} X^I \widehat{P}^A_{IB}) = 0 \,. \tag{3.21}$$

Using (2.10), (3.8) and (3.18) this equation is satisfied if $f^K_{IJ} = 0$, i.e. for abelian isometries.

It is known [38, 39, 40] that for scalar fields which obey eqs. (3.13) and (3.18) the Einstein equations corresponding to the the action (2.1) imply that the scalar potential V has to take the form

$$V = 6\left(\frac{3}{4}\, g^{MN} \partial_M W \partial_N W - W^2\right). \tag{3.22}$$

Indeed, inserting the special geometry relation $G^{IJ} = \partial_i X^I \partial_j X^J g^{ij} + \frac{2}{3}\, X^I X^J$, as well as eqs. (2.9) and (3.8), the relations (2.12), (3.19) and the definition (3.7) of W into (2.15) results in (3.22). Thus the special form of the potential (3.22) does not hold for an arbitrary $N = 2$ potential as given in (2.15) but requires precisely the same conditions that we needed to derive the flow equations.[6] This can be viewed as a consistency check on our procedure.

After this somewhat technical derivation let us summarize the results and discuss the physical implications. We solved the supersymmetric variations (3.3)–(3.5) in the background (3.1) and demanded four unbroken supersymmetries. This implies that only abelian isometries of the hypermultiplet geometry can be gauged, i.e. $f^K_{IJ} = 0, X^I k^i_I = 0$ with the further requirement $[\widehat{P}_I, \widehat{P}_J] = 0$. For the scalar fields a set of

[6]Strictly speaking eq. (3.22) does not need (3.8) but already holds for the weaker condition $[\partial_i X^I \widehat{P}_I, X^J \widehat{P}_J] = 0$. This relation is certainly satisfied for (3.8) but one could imagine a situation where the $\widehat{P}_I$ do not all commute but still satisfy $[\partial_i X^I \widehat{P}_I, X^J \widehat{P}_J] = 0$. However, this would put strong constraints on W, following from the very special geometry in 5-d which are not satisfied for standard choices of vector scalar manifolds.

first order differential equations follows. Provided (3.19) holds they can be written as gradient flow equations

$$\beta^M \equiv \mu \, \frac{d\Phi^M}{d\mu} = -3 \, g^{MN} \, \partial_N \log W \, , \tag{3.23}$$

where

$$W = \frac{2}{3} \left(X^I X^J \widehat{P}_I^x \widehat{P}_J^x \right)^{1/2} \; = \; \frac{2}{3} \, X^I \, \widehat{P}_I'^3 \, . \tag{3.24}$$

Eq. (3.23) combines eqs. (3.13) and (3.18) while the last equation in (3.24) uses the fact that the $\widehat{P}_I$ can always be chosen to point in the σ^3 direction.

The AdS/CFT correspondence suggests identifying the μ-derivative of the scalar fields with the β-function in the dual conformal field theory [8, 9, 10, 12, 13]. The fixed points of the RG-flow occur for $\partial_N W = 0$ which are also extrema of the scalar potential V as can be seen from eq. (3.22). For $W|_{\partial W=0} \neq 0$ the extremum corresponds to an AdS_5 background with W being the cosmological constant. $W|_{\partial W=0} = 0$ on the other hand corresponds to a flat space-time background.

The nature of the fixed point is determined by the derivatives of the β-functions or more precisely by the eigenvalues of the matrix

$$\partial_N \beta^M|_{\beta=0} = -3 \, g^{MK} \, \frac{\partial_N \partial_K W}{W} \bigg|_{\beta=0} \tag{3.25}$$

where we assume that the fixed point is non-singular, i.e. the metric is non-degenerate. Negative eigenvalues correspond to fixed points that are UV stable, while positive eigenvalues imply IR stable fixed points. A RG-flow configuration corresponds to a domain wall interpolating between a UV and a IR point, whereas a Randall-Sundrum type configuration interpolates between two IR fixed points, as we will discuss in the conclusions.

Let us briefly discuss a few generic cases. If there are no hypermultiplets in the spectrum the Killing prepotentials are constants, i.e. $P_I = \widehat{P}_I \equiv V_I =$ const. corresponding to Fayet-Illiopoulos terms. In this case eq. (3.24) implies

$$W = \frac{2}{3} \, X^I(\phi) \, V_I \, . \tag{3.26}$$

This form of W and the corresponding eqs. (3.22) and (3.23) reproduce the results derived previously in [29]. The very special geometry implies $\partial_i \partial_j W = \frac{2}{3} g_{ij} W$ and hence $\partial_i \beta^j|_{\beta=0} = -2\delta_i^j$. Thus all fixed points are necessarily ultraviolet [26, 29, 25]. In other words, neither RG-flows nor the RS scenario can be reproduced with only vector multiplets.[7]

If there are no vector multiplets but only hypermultiplets in the spectrum only the graviphoton can be used as a gauge field and the superpotential reduces to

$$W = \frac{2}{3} \, X^0 \, \widehat{P}_0'^3(q) \, . \tag{3.27}$$

[7] As shown in [25] this is also the case if some of the vectors are dualized into tensor multiplets.

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Its second derivative does not have a fixed sign and thus $\partial_u \beta^v|_{\beta=0}$ can have positive and/or negative eigenvalues.

In the case that vector- and hypermultiplets are present the matrix $\partial_N \beta^M|_{\beta=0}$ defined in (3.25) can have positive and negative eigenvalues. Negative eigenvalues are necessarily present since the submatrix of the vector multiplets $\partial_i \beta^j|_{\beta=0}$ always has negative eigenvalues as we have just discussed. Positive eigenvalues can arise from the derivatives in the direction of the hyperscalars $\partial_u \beta^v|_{\beta=0}$ but also from mixed derivatives of the form $\partial_i \beta^v|_{\beta=0}$. Thus any fixed point is a maximum or a saddle point but not a local minimum. Finally, note that for a superpotential that factorizes $W(q,\phi) = X(\phi)P(q)$ the mixed derivatives $\partial_i \beta^v|_{\beta=0}$ necessarily vanish. However, the possibility of $\partial_u \beta^v|_{\beta=0}$ having positive eigenvalues still remains and thus non-trivial DW solutions are also possible in this case.

In ref. [10] it was shown that whenever the scalar fields obey gradient flow equations of the type (3.23) the Einstein equations of the lagrangian (2.1) imply a c-theorem [29, 10, 8, 41] and that

$$C(\mu) = \frac{C_0}{|W|^3}\,, \qquad C_0 = \text{const.} \tag{3.28}$$

is a natural candidate for the c-function. This also holds in the setup here and eqs. (3.23) imply

$$\mu \frac{d}{d\mu} C = \frac{1}{|W|} g_{MN} \beta^M \beta^N > 0\,. \tag{3.29}$$

Thus C is a monotonically increasing function of μ and corresponds to the central charge at the conformal fixed points.

Before we turn to a specific example let us discuss the implications of the condition (3.8). The vanishing commutator together with (2.12) implies that all $P_I(q)$ are proportional to each other, i.e. $P_I(q) = \alpha_I(q) P(q)$. From eq. (2.12) we learn that also

$$\widehat{P}^A_{IB} = \widehat{\gamma}_I\, \alpha_I\, P^A_B \equiv \gamma_I(q)\, P^A_B(q) \tag{3.30}$$

holds. This in turn says that only a U(1) subgroup of the SU(2)$_R$ is gauged with a gauge field which is the (q-dependent) linear combination

$$A_m = \sum_I \gamma_I(q)\, A^I_m\,, \tag{3.31}$$

Similarly, a U(1) subgroup of the isometry group is gauged — albeit with a linear combination of gauge fields that differs in the q-dependent coefficients.

For constant γ_I, we recover precisely the case considered in refs. [27, 29, 31, 33] where only vector (and tensor) multiplets are present. With hypermultiplets we have the additional possibilty that the linear combination of gauge fields is q-dependent and thus can 'rotate' along an RG-flow. From the dual $N = 1$ field theory perspective this can be understood from the fact that the non-anomalous U(1)$_R$ symmetry also changes along an RG-flow [42, 43, 44, 45, 10]. Classically one has a $\mathrm{U}(1)_R$ (often denoted as the 'standard' $\mathrm{U}(1)_R$) which assigns zero R-charge to a chiral superfield and R-charge -1 to the field strength W_α of the vector multiplet. In addition, one generically has a flavour symmetry $\mathrm{U}(1)_K$ generated by the Konishi current which does transform the chiral multiplets but leaves W_α invariant [42, 43]. Quantum mechanically the situation changes in that the anomaly free $\mathrm{U}(1)_R$ is in general a linear combination of the standard $\mathrm{U}(1)_R$ with the $\mathrm{U}(1)_K$. The coefficents of this linear combination are related to the anomalous dimensions of certain operators and hence they change along an RG-flow. In the supergravity description this fact is captured by eq. (3.31). We return to this point in the next section.

4. Example of a BPS domain wall

In this section we discuss a specific BPS domain wall and show to what extent the solution of ref. [10] can be recovered. In [10] a DW of gauged $N = 8$ supergravity is given which interpolates between two AdS_5 vacua of the scalar potential V. One of the extrema preserves $N = 8$ supersymmetry while the second extrema only has $N = 2$ supersymmetry and the interpolating kink solution preserves $N = 1$ supersymmetry. In the AdS/CFT correspondence this BPS-solution is identified with a RG-flow from an $N = 4$ SCFT in the UV to an $N = 1$ SCFT in the IR which preserves $N = 1$ supersymmetry throughout the flow [45].

The gauge group of the $N = 8$ supergravity is $\mathrm{SO}(6) \sim \mathrm{SU}(4)$ which is identified with the R-symmetry of the $N = 4$ SCFT. This gauging introduces a scalar potential V which depends on the 42 scalars of the $N = 8$ gravitational multiplet spanning the coset $E_{6(6)}/\,\mathrm{USp}(8)$.

In order to simplify the analysis the authors of [10] decompose the gauge group according to $\mathrm{SU}(4) \to \mathrm{SU}(2)_I \times \mathrm{SU}(2)_G \times \mathrm{U}(1)_G$ and keep only $\mathrm{SU}(2)_I$ singlets. This corresponds to the breaking $N = 8 \to N = 4$ since 4 gravitini are projected out and $\mathrm{SU}(2)_G \times \mathrm{U}(1)_G$ becomes the gauged R-symmetry of the $N = 4$ supergravity. In the scalar sector 11 scalars which are the singlets of $\mathrm{SU}(2)_I$ survive this projection. It is shown that these 11 scalars span the coset

$$\mathcal{M} = \frac{\mathrm{SO}(5,2)}{\mathrm{SO}(5) \times \mathrm{SO}(2)} \times \mathrm{SO}(1,1)\,, \tag{4.1}$$

which is precisely the scalar manifold of two $N = 4$ tensor multiplets coupled to $N = 4$ supergravity [36]. An $N = 4$ tensor multiplet contains an antisymmetric tensor, four fermions and five scalars while the $N = 4$ gravitational multiplet contains the gravition, four gravitini, six graviphotons, four fermions and one scalar. This scalar spans the $\mathrm{SO}(1,1)$ component of $\mathcal{M}$.

In order to make contact with the previous section we need to do a further truncation to $N = 2$ along the lines of ref. [17]. This can be done by decomposing the gauge group further and again projecting onto invariant states. More precisely we decompose

$$\mathrm{SU}(2)_G \times \mathrm{U}(1)_G \to \mathrm{U}(1)_3 \times \mathrm{U}(1)_G\,, \tag{4.2}$$

where $\mathrm{U}(1)_3$ is the $\mathrm{U}(1)$ generated by σ^3 inside $\mathrm{SU}(2)_G$. We only keep states which are invariant under the diagonal subgroup of $\mathrm{U}(1)_3 \times \mathrm{U}(1)_G$. This leaves one $\mathrm{U}(1)_R$ (with the other combination of charges) intact.

The 8 gravitini of $N = 8$ supergravity transform as a $\mathbf{4} \oplus \bar{\mathbf{4}}$ of $\mathrm{SU}(4)$. In the decomposition $\mathrm{SU}(4) \to \mathrm{SU}(2)_I \times \mathrm{SU}(2)_G \times \mathrm{U}(1)_G$ the 4 $\mathrm{SU}(2)_I$ invariant gravitini transform according to the $\mathbf{2}_{-1/2} \oplus \bar{\mathbf{2}}_{1/2}$ of $\mathrm{SU}(2)_G \times \mathrm{U}(1)_G$. The $\mathrm{U}(1)_3 \times \mathrm{U}(1)_G$ invariance projects out two more gravitini leaving two complex conjugate gravitini transforming under $\mathrm{U}(1)_R$. The rest of the $N = 4$ spectrum can be similiarly truncated. Out of the four vectors of $\mathrm{SU}(2)_G \times \mathrm{U}(1)_G$ two abelian vectors of $\mathrm{U}(1)_3 \times \mathrm{U}(1)_G$ survive. The two tensors are both projected out while out of the 11 scalars 5 survive. The 5 scalars in the tensor multiplet reside in the representation $\mathbf{3}_1 \oplus \mathbf{1}_2 \oplus \mathbf{1}_0$ while the second tensor multiplet carries the complex conjugate representation. Thus after projection one is left with two singlets and two $\mathrm{U}(1)_R$ charged scalars. The 5th scalar comes out of the gravitational multiplet and is a singlet of $\mathrm{SU}(2)_G \times \mathrm{U}(1)_G$ and thus also of $\mathrm{U}(1)_R$. Out of the 12 fermions which reside in the $\mathbf{2}_{-3/2} \oplus \mathbf{2}_{+3/2} \oplus 2 \times (\mathbf{2}_{-1/2} \oplus \mathbf{2}_{+1/2})$ of $\mathrm{SU}(2)_G \times \mathrm{U}(1)_G$ four survive the projection. The surviving states fit precisely into one gravitational multiplet, one vector multiplet V and one hypermultiplet H of $N = 2$ supergravity. The scalar in V is neutral under $\mathrm{U}(1)_R$ while the hypermultiplet hosts the two neutral and the two charged scalars. The two neutral scalars of the hypermultiplet can be identified with the dilaton and axion of type-IIB supergravity.

The AdS/CFT correspondence relates these five scalars to gauge invariant operators in the dual CFT. The UV theory is an $N = 4$ SCFT with Yang-Mills gauge group $G = \mathrm{SU}(n)$. Written in terms of $N = 1$ superfields this theory has one vector multiplet, three chiral multiplet in the adjoint representation of G and a superpotential $W = \mathrm{Tr}\, A_1[A_2, A_3]$. The RG-flow is induced by adding the operator $m\,\mathrm{Tr}\, A_3^2$ to W [45, 10, 44]. The non-anomalous $\mathrm{U}(1)$ symmetry discussed at the end of section 3 is a linear combination of the $\mathrm{U}(1)_R$ at $m = 0$ which assign zero R-charge to all three superfields A and the Konishi $\mathrm{U}(1)_K$ symmetry which assigns A_3 a $\mathrm{U}(1)_K$ charge -1. In the dual supergravity the dilaton and axion play the role of the gauge coupling and the θ-angle, respectively. The charged scalar C couples to the operator $\mathrm{Tr}\, A_3^2$ and the five-dimensional vector couples to the Konishi current.

The resulting scalar manifold of the supergravity can be derived by truncating $\mathcal{M}$ given in eq. (4.1). The $\mathrm{SO}(1,1)$ factor of $\mathcal{M}$ survives the projection since its scalar is invariant. This component is a one-dimensional very special Kähler manifold characterized by [31]

$$\mathcal{V} = X^0 (X^1)^2 = 1\,. \tag{4.3}$$

The coset space $\mathrm{SO}(5,2)/(\mathrm{SO}(5)\times\mathrm{SO}(2))$ is a Kähler manifold with a Kähler potential

$$K = -\frac{1}{2}\ln\left[(S+\bar{S})(T+\bar{T}) - \frac{1}{2}\sum_{i=1}^{3}(C+\bar{C})_i^2\right]. \tag{4.4}$$

The three C_i are the triplet while S and T are singlets of $\mathrm{SU}(2)_G$. In addition one has $\mathrm{SL}(2,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})$ acting as fractional linear transformations on all fields. One $\mathrm{SL}(2,\mathbb{R})$ is the symmetry associated with the dilaton while the other $\mathrm{SL}(2,\mathbb{R})$ hosts the $\mathrm{U}(1)_G$ as its compact subgroup [10]. Thus projecting onto $\mathrm{U}(1)_3\times\mathrm{U}(1)_G$ invariant fields leaves S (or T) and one of the three C_i which henceforth, we denote by C. The Kähler potential becomes

$$K = -\frac{1}{2}\ln\left[(S+\bar{S}) - \frac{1}{2}(C+\bar{C})^2\right], \tag{4.5}$$

which is the Kähler potential of the coset space $\mathrm{SU}(2,1)/\,\mathrm{U}(2)$. This is indeed a quaternionic manifold known as the "universal hypermultiplet" [46]. Hence, the combined scalar manifold of the $N=2$ supergravity is found to be

$$\mathcal{M} = \mathrm{SO}(1,1)\times\frac{\mathrm{SU}(2,1)}{\mathrm{U}(2)}. \tag{4.6}$$

In the following we use a more convenient parameterization by shifting $S\to S+\frac{1}{2}C^2$ which results in

$$K = -\frac{1}{2}\ln\left[(S+\bar{S}) - C\bar{C}\right], \tag{4.7}$$

In these variables the $\mathrm{U}(1)_R$ acts as

$$C\to e^{i\theta}C\,, \qquad S\to S\,. \tag{4.8}$$

The next step is to gauge the isometry (4.8).[8] To do so we need to briefly recall the quaternionic quantities of $\mathrm{SU}(2,1)/\,\mathrm{U}(2)$ [46]. We use as quaternionic coordinates $q^u=(S,\bar{S},C,\bar{C})$. In these coordinates the $\mathrm{SU}(2)$ connection reads

$$\begin{aligned}
\omega_S &= \frac{1}{4}e^{2K}\sigma^3\,, & \omega_{\bar{S}} &= -\frac{1}{4}e^{2K}\sigma^3\,,\\
\omega_C &= \begin{pmatrix} -\frac{1}{4}e^{2K}\bar{C} & -e^K \\ 0 & \frac{1}{4}e^{2K}\bar{C}\end{pmatrix}, & \omega_{\bar{C}} &= \begin{pmatrix} \frac{1}{4}e^{2K}C & 0 \\ e^K & -\frac{1}{4}e^{2K}C\end{pmatrix}.
\end{aligned} \tag{4.9}$$

The matrix of hyper Kählerforms $(K_{uv})^A{}_B$ is given by

$$\begin{aligned}
K_{S\bar{S}} &= -\frac{1}{2}e^{4K}\sigma^3\,, & K_{C\bar{C}} &= \begin{pmatrix} \frac{1}{2}e^{2K}(1-e^{2K}C\bar{C}) & -e^{3K}C \\ -e^{3K}\bar{C} & -\frac{1}{2}e^{2K}(1-e^{2K}C\bar{C})\end{pmatrix}\\
K_{S\bar{C}} &= \begin{pmatrix} \frac{1}{2}e^{4K}C & 0 \\ e^{3K} & -\frac{1}{2}e^{4K}C\end{pmatrix}, & K_{\bar{S}C} &= \begin{pmatrix} -\frac{1}{2}e^{4K}\bar{C} & -e^{3K} \\ 0 & \frac{1}{2}e^{4K}\bar{C}\end{pmatrix},
\end{aligned} \tag{4.10}$$

[8] In ref. [27] the same quaternionic geometry was considered but a different isometry corresponding the shift $S\to S+\alpha, C\to C$ was gauged. This leads to a different potential which does not correspond to a RG-type domain wall.

while all other components are zero. The Killing vector for the symmetry (4.8) is

$$k^u = \big(0, 0, i\,C, -i\,\bar{C}\big)\,. \tag{4.11}$$

Using (4.9)–(4.11) the solution of eq. (2.9) is found to be

$$P^A_{\ B} = \frac{i}{2}\begin{pmatrix} 1 - e^{2K} C\bar{C} & -2e^K\,C \\ -2e^K\,\bar{C} & -(1 - e^{2K} C\bar{C}) \end{pmatrix}. \tag{4.12}$$

As we stated above the $\mathrm{U}(1)_R$ is a linear combination of the $\mathrm{U}(1)_3$ and the $\mathrm{U}(1)_G$ and therefore both gauge fields appear in the covariant derivatives. Using (3.31) we allow this linear combination to be q-dependent and from (3.30) we infer that the $\widehat{P}$ obey

$$\widehat{P}_0 = \gamma_0 P\,, \qquad \widehat{P}_1 = \gamma_1 P\,, \tag{4.13}$$

where P is given by (4.12) and the γ_I satisfy (3.20). Inserted into (3.24) the resulting superpotential is

$$W = \frac{1}{3}\,(\gamma_0 X^0 + \gamma_1 X^1)\,\frac{S + \bar{S}}{S + \bar{S} - C\bar{C}}\,. \tag{4.14}$$

Using the constraint (4.3) and introducing new variables

$$X^1 \equiv \rho^{-2}\,, \qquad \frac{C\bar{C}}{S + \bar{S}} \equiv \tanh^2(\chi)\,, \tag{4.15}$$

yields

$$W = \frac{1}{3\rho^2}\,(\gamma_0\,\rho^6 + \gamma_1)\,\cosh^2(\chi)\,. \tag{4.16}$$

For the choice

$$\gamma_0 = \frac{3}{2}(2\tanh^2(\chi) - 1)\,, \qquad \gamma_1 = -3\,, \tag{4.17}$$

one obtains

$$W = \frac{1}{4\rho^2}\,\big[(\rho^6 - 2)\cosh(2\chi) - (3\rho^6 + 2)\big]\,, \tag{4.18}$$

which precisely coincides with the superpotential of ref. [10]. The RG-flow governed by this superpotential has a UV fixed point at $\rho = 1$, $\chi = 0$ and an IR fixed point at $\rho^6 = 2$, $2\chi = \log 3$. It is important to note that with constant γ_0, W factorizes as can be seen from (4.16) and it is not possible to recover (4.18). Thus it is crucial to allow for q-dependent γ_0 and this is the main motivation for introducing $\widehat{\gamma}_I$ in eq. (2.12). The specific γ_0 of (4.17) indeed satisfies the constraint (3.20) for $\alpha_0 = 3/2$, $\alpha_1 = -3$.

Finally let us note that the dilaton S automatically stays constant along this flow. Using (3.23), (4.5), (4.14) and (4.17) one finds

$$\mu\frac{d}{d\mu}S = -\frac{3}{W}(g^{S\bar{S}}\partial_{\bar{S}}W + g^{S\bar{C}}\partial_{\bar{C}}W) = 0\,. \tag{4.19}$$

This can be viewed as a consistency check of our solution. Note that for the universal hypermultiplet a constant dilaton along the RG-trajectory is not a special feature of (4.18) but holds for any superpotential W which is a function of $\frac{C\bar{C}}{S+\bar{S}}$ only as can be easily verified from eq. (4.19).

5. Conclusions

In this paper we derived $N = 1$ BPS domain wall solutions of gauged five-dimensional $N = 2$ supergravity. Our main result is the derivation of the supersymmetric flow equations (3.23), (3.24) which include scalars from vector and hypermultiplets. The presence of charged hypermultiplets turns out to be crucial in recovering IR-fixed points in RG-flows of a dual (perturbed) superconformal field theory. In order to recover the specific flow of ref. [10] it is necessary to modify the standard gauging of the R-symmetry. The validity of this modification remains to be rigorously proven. However, the fact that we do recover the flow of [10] can also be viewed as a consistency check.

The necessity of IR fixed points in order to construct a smooth supersymmetric domain-wall solution of the Randall-Sundrum type has been stressed in ref. [25]. Let us briefly recall the argument. It is convenient to first change coordinates and replace the μ of the ansatz (3.1) by

$$\mu = e^{A(z)}\,, \qquad W = \partial_z A(z)\,. \tag{5.1}$$

In these coordinates the metric (3.1) reads

$$ds^2 = e^{2A(z)}\Big(- dt^2 + d\vec{x}^2\Big) + dz^2\,, \tag{5.2}$$

which is the metric of AdS_5 for $A = \pm kz, k = \text{const}$. In these coordinates the UV fixed point ($\mu \to \infty$) of a RG-flow is located at $z \to \infty$ where $A \sim z \to +\infty$ while the IR fixed point ($\mu = 0$) sits at $z \to -\infty$ where $A \sim z \to -\infty$. The DW solution interpolates between the two asymptotic regions at $z = \pm\infty$.

A smooth DW solution corresponding to the RS-setup needs to have a different asymptotic behavior. In that case one has a $\mathbb{Z}_2$ symmetric solution with $A \to -k|z|$ for $z \to \pm\infty$ [18]. That is one has a decreasing warp factor at both ends $z \to \pm\infty$ or in the language of the RG-flow a DW solution connecting two IR-fixed points [25]. Obviously, such a solution cannot be interpreted as an RG-flow. $A(z)$ has at least one maximum where $W = \partial A(z) = 0$. At that point the β-functions of eqs. (3.23) as well as the c-function of (3.28) become singular [47]. However, even if such DW solutions do not make sense as RG-flows there is no obvious reason why they should not exist. The previous no-go theorems merely stated that they cannot be found with only non-trivial vector and tensor multiplets. Adding charged hypermultiplets changes the story and we are optimistic that supersymmetric RS-domain walls do exist [48]. They should be smooth generalizations of the constructions presented in refs. [19]–[24].

Finally, IR-fixed points have also been recently studied by wrapping M5-branes on a Riemann surface of constant negative curvature in the presence of a non-trivial gauge field [49]. Following this procedure, one obtains after compactification to 5 dimensions a 3-brane solution with an AdS_5 vacuum, which is IR attractive (near the AdS horizon). But in the UV limit this solution decompactifies into AdS_7, i.e. is singular from the 5-dimensional perspective. It would be interesting to study this situation within the formalism of this paper.

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References

[1] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rep.* **323** (2000) 183 [hep-th/9905111].

[2] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[3] S. Kachru and E. Silverstein, *4d conformal theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183].

[4] Y. Oz and J. Terning, *Orbifolds of $AdS_5 \times S^5$ and 4d conformal field theories*, *Nucl. Phys.* **B 532** (1998) 163 [hep-th/9803167].

[5] I.R. Klebanov and E. Witten, *Superconformal field theory on threebranes at a Calabi-Yau singularity*, *Nucl. Phys.* **B 536** (1998) 199 [hep-th/9807080];
AdS/CFT correspondence and symmetry breaking, *Nucl. Phys.* **B 556** (1999) 89 [hep-th/9905104].

[6] B.S. Acharya, J.M. Figueroa-O'Farrill, C.M. Hull and B. Spence, *Branes at conical singularities and holography*, *Adv. Theor. Math. Phys.* **2** (1999) 1249 [hep-th/9808014].

[7] D.R. Morrison and M.R. Plesser, *Non-spherical horizons, 1*, *Adv. Theor. Math. Phys.* **3** (1999) 1 [hep-th/9810201].

[8] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Novel local CFT and exact results on perturbations of $N = 4$ super Yang-Mills from AdS dynamics*, *J. High Energy Phys.* **12** (1998) 022 [hep-th/9810126].

[9] J. Distler and F. Zamora, *Non-supersymmetric conformal field theories from stable anti-de Sitter spaces*, *Adv. Theor. Math. Phys.* **2** (1999) 1405 [hep-th/9810206].

[10] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Renormalization group flows from holography supersymmetry and a c-theorem*, hep-th/9904017.

[11] A. Khavaev, K. Pilch and N.P. Warner, *New vacua of gauged $N = 8$ supergravity in five dimensions*, *Phys. Lett.* **B 487** (2000) 14 [hep-th/9812035].

[12] J. de Boer, E. Verlinde and H. Verlinde, *On the holographic renormalization group*, *J. High Energy Phys.* **08** (2000) 003 [hep-th/9912012].

[13] C. Schmidhuber, *AdS-flows and Weyl gravity*, *Nucl. Phys.* **B 580** (2000) 121 [hep-th/9912155].

[14] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[15] A.W. Peet and J. Polchinski, *UV/IR relations in AdS dynamics*, *Phys. Rev.* **D 59** (1999) 065011 [hep-th/9809022].

[16] K. Pilch and N.P. Warner, *$N = 2$ supersymmetric RG flows and the IIB dilaton*, *Nucl. Phys.* **B 594** (2001) 209 [hep-th/0004063].

[17] K. Pilch and N.P. Warner, *$N = 1$ supersymmetric renormalization group flows from IIB supergravity*, hep-th/0006066.

[18] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064]; *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[19] R. Altendorfer, J. Bagger and D. Nemeschansky, *Supersymmetric randall-sundrum scenario*, hep-th/0003117.

[20] T. Gherghetta and A. Pomarol, *Bulk fields and supersymmetry in a slice of AdS*, *Nucl. Phys.* **B 586** (2000) 141 [hep-ph/0003129].

[21] A. Falkowski, Z. Lalak and S. Pokorski, *Supersymmetrizing branes with bulk in five-dimensional supergravity*, *Phys. Lett.* **B 491** (2000) 172 [hep-th/0004093].

[22] E. Bergshoeff, R. Kallosh and A.V. Proeyen, *Supersymmetry in singular spaces*, *J. High Energy Phys.* **10** (2000) 033 [hep-th/0007044].

[23] M.J. Duff, J.T. Liu and K.S. Stelle, *A supersymmetric type-IIB Randall-Sundrum realization*, hep-th/0007120.

[24] M. Cvetič, H. Lu and C.N. Pope, *Domain walls with localised gravity and domain-wall/QFT correspondence*, hep-th/0007209.

[25] R. Kallosh and A. Linde, *Supersymmetry and the brane world*, *J. High Energy Phys.* **02** (2000) 005 [hep-th/0001071].

[26] K. Behrndt and M. Cvetič, *Anti-de Sitter vacua of gauged supergravities with 8 supercharges*, *Phys. Rev.* **D 61** (2000) 101901 [hep-th/0001159].

[27] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Heterotic M-theory in five dimensions*, *Nucl. Phys.* **B 552** (1999) 246 [hep-th/9806051]; *The universe as a domain wall*, *Phys. Rev.* **D 59** (1999) 086001 [hep-th/9803235].

[28] J. Ellis, Z. Lalak and W. Pokorski, *Five-dimensional gauged supergravity and supersymmetry breaking in M-theory*, *Nucl. Phys.* **B 559** (1999) 71 [hep-th/9811133].

[29] K. Behrndt, *Domain walls of $D = 5$ supergravity and fixpoints of $N = 1$ super Yang-Mills*, *Nucl. Phys.* **B 573** (2000) 127 [hep-th/9907070].

[30] K. Behrndt and M. Cvetič, *Supersymmetric domain wall world from $D = 5$ simple gauged supergravity*, *Phys. Lett.* **B 475** (2000) 253 [hep-th/9909058].

[31] M. Gunaydin and M. Zagermann, *The gauging of five-dimensional, $N = 2$ maxwell-einstein supergravity theories coupled to tensor multiplets*, *Nucl. Phys.* **B 572** (2000) 131 [hep-th/9912027].

[32] M. Gunaydin and M. Zagermann, *The vacua of 5d, $N = 2$ gauged Yang-Mills/Einstein/tensor supergravity: abelian case*, *Phys. Rev.* **D 62** (2000) 044028 [hep-th/0002228].

[33] A. Ceresole and G. Dall'Agata, *General matter coupled $N = 2$, $D = 5$ gauged supergravity*, *Nucl. Phys.* **B 585** (2000) 143 [hep-th/0004111].

[34] M. Cvetič, S. Griffies and S.-J. Rey, *Static domain walls in $N = 1$ supergravity*, *Nucl. Phys.* **B 381** (1992) 301 [hep-th/9201007];
M. Cvetič and H.H. Soleng, *Supergravity domain walls*, *Phys. Rep.* **282** (1997) 159 [hep-th/9604090].

[35] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D'Auria, S. Ferrara and P. Fré, *General matter coupled $N = 2$ supergravity*, *Nucl. Phys.* **B 476** (1996) 397 [hep-th/9603004];
L. Andrianopoli, M. Bertolini, A. Ceresole, R. D'Auria, S. Ferrara, P. Fré and T. Magri, *$N = 2$ supergravity and $N = 2$ super Yang-Mills theory on general scalar manifolds: symplectic covariance, gaugings and the momentum map*, *J. Geom. Phys.* **23** (1997) 111 [hep-th/9605032].

[36] M. Gunaydin, L.J. Romans and N.P. Warner, *Compact and noncompact gauged supergravity theories in five-dimensions*, *Nucl. Phys.* **B 272** (1986) 598.

[37] M. Gunaydin, G. Sierra and P.K. Townsend, *Gauging the $D = 5$ Maxwell-Einstein supergravity theories: more on Jordan algebras*, *Nucl. Phys.* **B 253** (1985) 573.

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[38] P.K. Townsend, *Positive energy and the scalar potential in higher dimensional (super)gravity theories*, *Phys. Lett.* **B 148** (1984) 55.

[39] K. Skenderis and P.K. Townsend, *Gravitational stability and renormalization-group flow*, *Phys. Lett.* **B 468** (1999) 46 [hep-th/9909070].

[40] A. Chamblin and G.W. Gibbons, *Nonlinear supergravity on a brane without compactification*, *Phys. Rev. Lett.* **84** (2000) 1090 [hep-th/9909130].

[41] E. Alvarez and C. Gomez, *Geometric holography, the renormalization group and the c-theorem*, *Nucl. Phys.* **B 541** (1999) 441 [hep-th/9807226].

[42] N. Seiberg, *Electric-magnetic duality in supersymmetric nonabelian gauge theories*, *Nucl. Phys.* **B 435** (1995) 129 [hep-th/9411149].

[43] I.I. Kogan, M. Shifman and A. Vainshtein, *Matching conditions and duality in $N = 1$ SUSY gauge theories in the conformal window*, *Phys. Rev.* **D 53** (1996) 4526 [hep-th/9507170].

[44] R.G. Leigh and M.J. Strassler, *Exactly marginal operators and duality in four-dimensional $N = 1$ supersymmetric gauge theory*, *Nucl. Phys.* **B 447** (1995) 95 [hep-th/9503121].

[45] A. Karch, D. Lüst and A. Miemiec, *New $N = 1$ superconformal field theories and their supergravity description*, *Phys. Lett.* **B 454** (1999) 265 [hep-th/9901041].

[46] S. Ferrara and S. Sabharwal, *Quaternionic manifolds for type-II superstring vacua of Calabi-Yau spaces*, *Nucl. Phys.* **B 332** (1990) 317.

[47] G.W. Gibbons and N.D. Lambert, *Domain walls and solitons in odd dimensions*, *Phys. Lett.* **B 488** (2000) 90 [hep-th/0003197].

[48] K. Behrndt, C. Herrmann, J. Louis and S. Thomas, in preparation.

[49] J. Maldacena and C. Nuñez, *Supergravity description of field theories on curved manifolds and a no go theorem*, hep-th/0007018.